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# Higher Mathematics (2014 on)

## Expressions and Functions

## Practice Unit Assessment C

**Notes:**

1. Read the question fully before answering it.
2. Always show your working.
3. Check your paper at the end if you have time.

# FORMULAE SHEET

**Scalar Product:** **a.b** = |**a**||**b**| $\cos\theta$ , where $\theta$ is the angle between **a** and **b**.

or **a.b** $= a_1b_1 + a_2b_2 + a_3b_3$ where **a** $= \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and **b** $= \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

**Trigonometric formulae:** $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\sin 2A = 2\sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

**Answer all the questions**

**E&F Assessment Standard 1.1**

**1.** (a) Simplify $\log_2 7c + \log_2 3d$. **(1)**

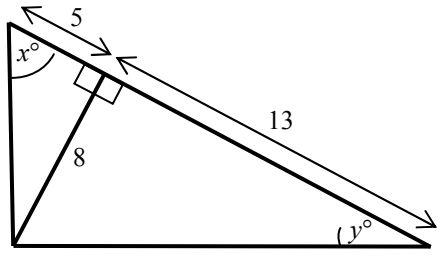
(b) Express $\log_n b^{10} - \log_n b^3$ in the form $k \log_n b$. **(2)**

**2.** Solve $\log_2(x-8) = 5$. **(2)**

**E&F Assessment Standard 1.2**

**3.** Express $2\sin x - 4\cos x$ in the form $k\sin(x-a)°$ where $k > 0$ and $0 \le a < 360$. **(4)**

**4.** The diagram below shows two right-angled triangles.
Find the exact value of $\sin(x-y)$.



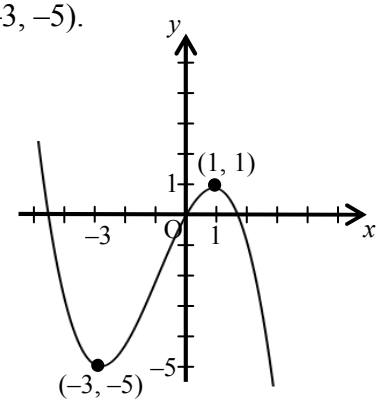

**(4)**

**5.** Show that $(5-4\sin x)(5+4\sin x) = 16\cos^2 x + 9$. **(3)**
**(#2.1)**

**E&F Assessment Standard 1.3**

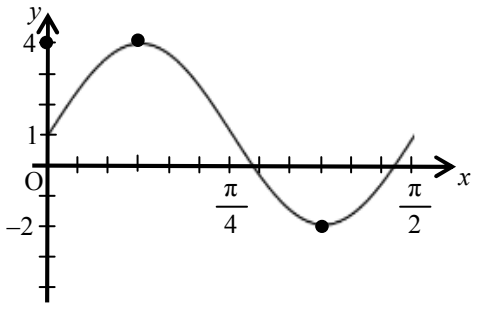
**6.** Sketch the graph of $y = a\cos\left(x + \frac{\pi}{6}\right)$ for $0 \le x \le 2\pi$ and $a > 0$, clearly showing the maximum and minimum values and where it cuts the *x*-axis. **(3)**

**7.** The diagram shows the graph of $y = f(x)$ with a maximum turning point at (1, 1) and a minimum turning point at (–3, –5).



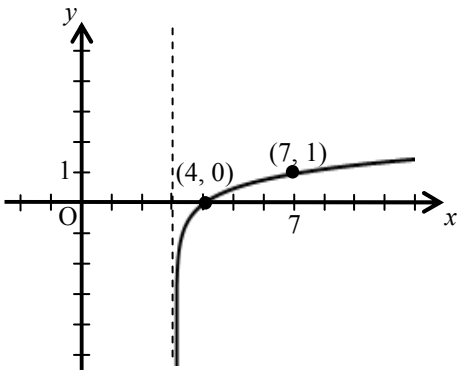

Sketch the graph of $y = f(x-3) + 2$. **(3)**

**8.** The diagram shows the graph of $y = a\sin(bx) + c$.




Write down the values of *a*, *b* and *c*. **(3)**

**9.** The diagram shows the graph of $y = \log_b(x - a)$.




Determine the values of *a* and *b*. **(2)**

**10.** The functions *f* and *g*, defined on suitable domains, are given by $f(x) = 5x + 9$ and $g(x) = \dfrac{2}{1+x}$. A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(a) Find an expression for $h(x)$. **(2)**

(b) Show that the domain for $h(x)$ is given by $x \neq -2$ and explain why this is. **(1)**

**(#2.2)**

**11.** A function is given by $f(x) = 7x + 1$. Find the inverse function $f^{-1}(x)$. **(2)**

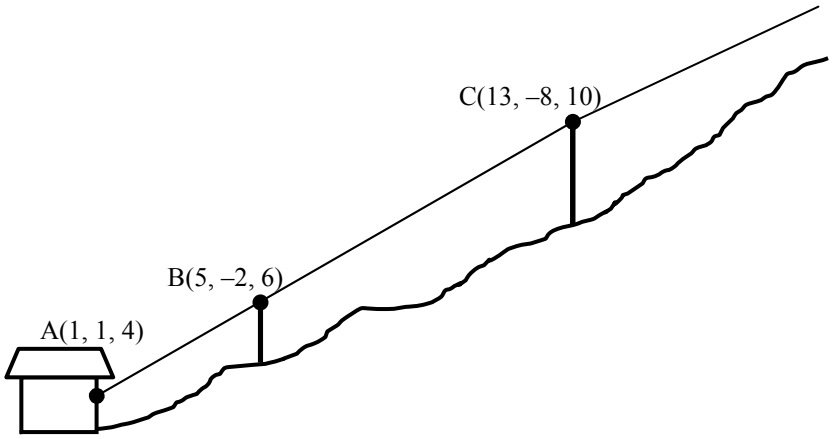
**E&F Assessment Standard 1.4**

**12.** An engineer is designing a cable car for a tourist resort so that visitors can get to a restaurant high up a hillside.

In order to give the cable car a smooth start he is working to the following conditions:

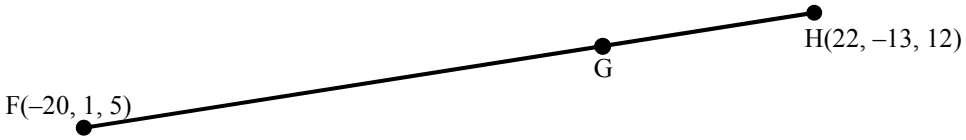
- the first two pylons must follow a straight line from the base station;
- the distance between the first and second pylons is double the distance from the base station to the first pylon.

Relative to a suitable axes, the base station can be represented as A(1, 1, 4) and the top of each pylon can be represented by the points B(5, –2, 6) and C(13, –8, 10).




Have the pylons been spaced correctly? You must justify your answer. **(4)** (#2.1) (#2.2)

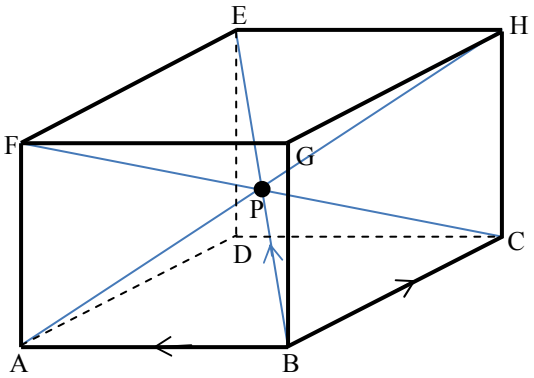
**13.** The points F, G and H lie in a straight line, as shown. G divides FH in the ratio 5:2.




Find the coordinates of G. **(3)**

**Turn over for questions 14 and 15**

**14.** ABCDEFGH is a cuboid with centre P.




The vectors $\overrightarrow{BA}$, $\overrightarrow{BC}$ and $\overrightarrow{BP}$ are given by:

$\overrightarrow{BA} = -6\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$

$\overrightarrow{BC} = 2\mathbf{i} + 10\mathbf{j} + 8\mathbf{k}$

$\overrightarrow{BP} = -3\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$

Express $\overrightarrow{CP}$ in component form. **(3)**

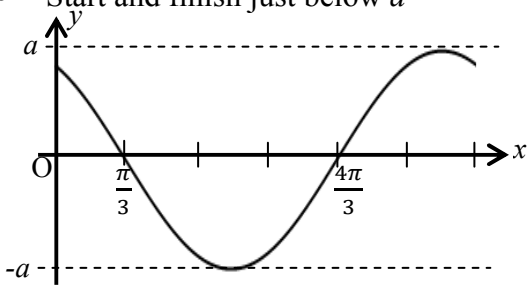
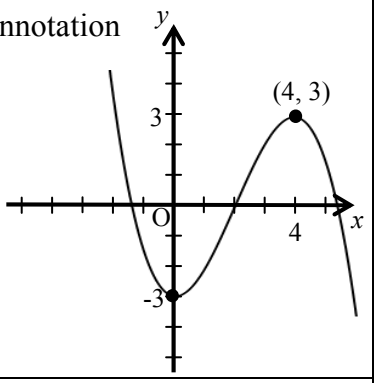
**15.** Points P, Q and R have coordinates A(1, 5, –2), B(–5, 7, –1) and C(4, 7, 1).


Find the size of the obtuse angle ABC. **(5)**

**End of question paper**

**Expressions and Functions, Practice Assessment C**

| Qu. | Points of Expected Response | Illustrative Scheme |
|---|---|---|
| **E&F Outcome 1.1: Logs and Exponentials** | | |
| 1(a) | •[1] Use $\log_a x + \log_a y = \log_a xy$ | •[1] $\log_2 21cd$ |
| 1(b) | •[2] Use $\log_a x - \log_a y = \log_a \frac{x}{y}$ OR $\log_a x^m = m \log_a x$<br>•[3] Simplify to $k \log_a x$ | •[2] $\log_n b^7$ OR $10 \log_n b - 3 \log_n b$<br>•[3] $7\log_n b$ |
| 2 | •[1] Start to solve<br>•[2] Solve correctly | •[1] $(x-8) = 2^5$<br>•[2] $x = 40$<br>**5** |
| **E&F Outcome 1.2: Trigonometry** | | |
| 3 | •[1] Expand $k \sin(x-a)$<br>•[2] Compare coefficients<br>•[3] Process $k$<br>•[4] Process $a$ | •[1] $k \sin x \cos a - k \cos x \sin a$ stated explicitly<br>•[2] $k \cos a = 2$ **and** $k \sin a = 4$ stated explicitly<br>•[3] $k = \sqrt{20}$ or equivalent<br>•[4] $a = 63 \cdot 4°$ |
| 4 | •[1] Process missing sides<br>•[2] Expand $\sin(x-y)$ **and** begin subs<br>•[3] Complete substitution<br>•[4] Process | •[1] $\sqrt{89}$ and $\sqrt{233}$<br>•[2] $\frac{8}{\sqrt{89}} \cos y - \cos x \sin y$<br>•[3] $\frac{8}{\sqrt{89}} \times \frac{13}{\sqrt{233}} - \frac{5}{\sqrt{89}} \times \frac{8}{\sqrt{233}}$<br>•[4] $\frac{64}{\sqrt{20737}}$ or equivalent |
| 5 | •[1] Expand LHS<br>•[2] Substitute for $\sin^2 x$<br>•[3] Simplify and complete<br>#[2.1] Valid strategy | •[1] $25 - 16 \sin^2 x$<br>•[2] $25 - 16(1 - \cos^2 x)$<br>•[3] $16 \cos^2 x + 9$<br>#[2.1] Know to use identity<br>**11 + #[2.1]** |

| Qu. | Points of Expected Response | Illustrative Scheme |
|---|---|---|
| **E&F Outcome 1.3: Trigonometry** | | |
| 6 | •[1] Max/min correct<br>•[2] $x$-intercepts<br>•[3] Correct shape<br><br>**Note:**<br>The y-intercept is not required. | •[1] On graph max value $a$ and min value $-a$<br>•[2] $\frac{\pi}{3}$ and $\frac{4\pi}{3}$<br>•[3] Start and finish just below $a$<br>  |
| 7 | •[1] Correct horizontal translation<br>•[2] Correct vertical translation<br>•[3] Correctly annotated diagram<br><br>**Notes:**<br>For correctly drawn and annotated graphs of $y = f(x+3)+2$ or $y = f(x-3)-2$ award 2 points of process.<br>For $y = f(x-3)+a$ $(a \neq 2)$ where $x$-coordinates are consistent award 1 point.<br>For $y = f(x-a)+2$ $(a \neq 3)$ where y-coordinates are consistent award 1 point. | •[1] $x$-coordinates correct<br>•[2] $y$-coordinates correct<br>•[3] Correct shape and annotation<br>  |
| 8 | •[1] Find $a$<br>•[2] Find $b$<br>•[3] Find $c$ | •[1] $a = 3$<br>•[2] $b = 4$<br>•[3] $c = 1$ |
| 9 | •[1] Find $a$<br>•[2] Find $b$ | •[1] $a = 3$<br>•[2] $b = 4$ |
| 10a | •[1] Interpret composite process<br>•[2] Complete function | •[1] $h(x) = g(5x+9)$<br>•[2] $h(x) = \frac{2}{5x+10}$ |
| 10b | •[3] Interpret domain of $h(x)$<br>#[2.2] Explain a solution | •[3] $5x + 10 \geq 0$ leading to $x \geq -2$<br>#[2.2] In real numbers the square root of a negative number cannot be found. |
| 11 | •[1] Start inverse process<br>•[2] State inverse function | •[1] $y - 1 = 7x$<br>•[2] $f^{-1}(x) = \frac{x-1}{7}$<br>**16 + #[2.2]** |

| Qu. | Points of Expected Response | Illustrative Scheme |
|---|---|---|
| **E&F Outcome 1.4: Vectors** | | |
| 12 | $\#^{2.1}$ Strategy<br>$\bullet^1$ Find vector between two points<br>$\bullet^2$ Find second vector and interpret multiple<br>$\bullet^3$ Complete proof of collinearity<br>$\bullet^4$ Interpret ratio<br>$\#^{2.2}$ Explain solution in context | $\#^{2.1}$ Show points are not collinear<br>$\bullet^1$ e.g. $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}$<br>$\bullet^2$ e.g. $\overrightarrow{BC} = \begin{pmatrix} 8 \\ -6 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}$<br>$\bullet^3$ $\overrightarrow{AB} = 2\overrightarrow{BC}$ hence the vectors are parallel. B is a common point so the points A, B and C are collinear.<br>$\bullet^4$ $\overrightarrow{AB}$: $\overrightarrow{BC}$ are in the ratio 1:2<br>$\#^{2.2}$ The pylons have been set out correctly as both conditions have been met. |
| 13 | $\bullet^1$ Find component vector $\overrightarrow{FH}$<br>$\bullet^2$ Use correct ratio<br>$\bullet^3$ Process vectors and find point G<br>*Alternative Method: Section formula*<br>$\bullet^1$ Begin substitution<br>$\bullet^2$ Complete substitution<br>$\bullet^3$ Process and state point G | $\bullet^1$ $\overrightarrow{FH} = \begin{pmatrix} 42 \\ -14 \\ 7 \end{pmatrix}$<br>$\bullet^2$ $\overrightarrow{FG} = \frac{5}{7}\begin{pmatrix} 27 \\ 18 \\ -9 \end{pmatrix}$<br>$\bullet^3$ G = (10, –9, 10)<br>*Alternative Method: Section formula*<br>$\bullet^1$ $\boldsymbol{g} = \frac{2\boldsymbol{f}+5\boldsymbol{h}}{9}$<br>$\bullet^2$ $\boldsymbol{g} = \frac{2\begin{pmatrix} -20 \\ 1 \\ 5 \end{pmatrix}+5\begin{pmatrix} 22 \\ -13 \\ 12 \end{pmatrix}}{7}$<br>$\bullet^3$ G = (10, –9, 10) |
| 14 | $\bullet^1$ Find pathway for $\overrightarrow{CP}$<br>$\bullet^2$ Identify $-\overrightarrow{BP}$<br>$\bullet^3$ Complete calculation of $\overrightarrow{CP}$ | $\bullet^1$ $\overrightarrow{CP} = -\overrightarrow{BC} + \overrightarrow{BP}$<br>$\bullet^2$ $-(-3\mathbf{i} + 5\mathbf{j} + 5\mathbf{k})$<br>$\bullet^3$ $-5\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}$ OR $\begin{pmatrix} -5 \\ -5 \\ -3 \end{pmatrix}$<br>Do not award $\bullet^3$ for (–5 , –5, –3) |
| 15 | $\bullet^1$ Find component vectors<br>$\bullet^2$ Use scalar product<br>$\bullet^3$ Process scalar product<br>$\bullet^4$ Process magnitudes<br>$\bullet^5$ Find angle | $\bullet^1$ $\overrightarrow{AB} = \begin{pmatrix} -6 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$<br>$\bullet^2$ $\cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert}$<br>$\bullet^3$ –11<br>$\bullet^4$ $\sqrt{40}$ and $\sqrt{41}$<br>$\bullet^5$ $\theta$ = 105·8° or 1·85 (radians)<br>**15 + $\#^{2.1}$ + $\#^{2.2}$** |